OSCILLATIONS

LET'S PLAY WITH PHYSICS—NEET FORMULA SHEET

1 Basic SHM Definitions

Condition for SHM: $a = -\omega^2 x$

Displacement:

$$x(t) = A\sin(\omega t + \phi) \quad \text{or} \quad x(t) = A\cos(\omega t + \phi)$$

Angular frequency:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Time period & frequency:

$$T = \frac{2\pi}{\omega}, \qquad f = \frac{1}{T}$$

Reading from equation $x = A\sin(\omega t + \phi)$:

- Amplitude $= A$
- $T = \frac{2\pi}{\omega}$
- Initial phase $= \phi$

PYQ Trick (2024, 2019):
$x = 5\sin(\pi t + \frac{\pi}{3})$ m $\Rightarrow A = 5$ m, $T = \frac{2\pi}{\pi} = 2$ s

2 Velocity & Acceleration in SHM

$$v = \omega\sqrt{A^2 - x^2}$$

$$v_{\max} = \omega A \quad (\text{at } x = 0, \text{ mean position})$$

$$a = -\omega^2 x$$

$$a_{\max} = \omega^2 A \quad (\text{at } x = \pm A, \text{ extreme})$$

Phase relations:

- v leads x by $\frac{\pi}{2}$
- a is exactly **opposite** (phase diff $= \pi$) to x

NEET 2020 PYQ: Phase difference between displacement and acceleration $= \boldsymbol{\pi}$ rad *(not $\pi/2$!)*

v–x relation (ellipse):

$$\frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1$$

Velocity-displacement graph is an **ellipse**.

3 Energy in SHM

$$KE = \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$PE = \frac{1}{2}m\omega^2 x^2$$

$$E_{total} = \frac{1}{2}m\omega^2 A^2 = \text{constant}$$

- At $x = 0$ (mean): $KE = E_{total}$, $PE = 0$
- At $x = A$ (extreme): $KE = 0$, $PE = E_{total}$
- At $x = \frac{A}{\sqrt{2}}$: $KE = PE = \frac{E}{2}$

NEET 2021 PYQ (most repeated):
If SHM frequency is n, frequency of KE or $PE = \mathbf{2n}$
(Energy completes 2 cycles per oscillation)

PE when particle is at $x = A/2$:

$$PE = \frac{1}{2}m\omega^2\left(\frac{A}{2}\right)^2 = \frac{1}{4}\cdot\frac{1}{2}m\omega^2 A^2 = \frac{E_{total}}{4}$$

4 Spring–Mass System

$$T = 2\pi\sqrt{\frac{m}{k}}, \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{k}{m}}$$

Springs in series:

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} \quad \Rightarrow \quad k_{eff} = \frac{k_1 k_2}{k_1 + k_2}$$

Springs in parallel:

$$k_{eff} = k_1 + k_2$$

Cutting a spring (PYQ 2021):
If spring of constant k is cut into ratio $1:2:3$,
each piece has constant $= k \times \frac{\text{total length}}{\text{piece length}}$
e.g. piece of length $L/6 \Rightarrow k' = 6k$

Series combination:

$$T_s = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

Parallel combination:

$$T_p = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

5 Simple Pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Effect of acceleration of frame:

$$T' = 2\pi\sqrt{\frac{L}{g_{eff}}}$$

- Lift accelerating up: $g_{eff} = g + a$ (T **decreases**)
- Lift accelerating down: $g_{eff} = g - a$ (T **increases**)
- Free fall: $g_{eff} = 0$ ($T \to \infty$, no oscillation)

On moon: $g_{moon} = g/6 \Rightarrow T_{moon} = \sqrt{6}\,T_{earth}$

Bob velocity at mean position:

$$v = \sqrt{2gL(1-\cos\theta)} = \sqrt{2gL} \cdot \sin\left(\frac{\theta}{2}\right) \cdot \sqrt{2}$$

Simplified: $v = \sqrt{2gL(1-\cos\theta)}$

NEET 2023 PYQ (pendulum in liquid):
If liquid density $= \rho_l$ and bob density $= \rho_b$:

$$g_{eff} = g\left(1 - \frac{\rho_l}{\rho_b}\right)$$

6 Superposition of SHMs

If $y = A\sin\omega t + B\cos\omega t$:

$$\text{Resultant Amplitude} = \sqrt{A^2 + B^2}$$

If $y = A_0 + A\sin\omega t + B\cos\omega t$:

Amplitude $= \sqrt{A^2 + B^2}$ (A_0 shifts mean, not amplitude)

NEET 2019 PYQ (most repeated):
$y = A_0 + A\sin\omega t + B\cos\omega t$
Amplitude $= \sqrt{A^2 + B^2}$ (**NOT** $A_0 + \sqrt{A^2 + B^2}$)

Two SHMs at right angles with phase diff $\pi/2$:
Motion is **circular** if amplitudes are equal.

7 Time Calculations in SHM

Using $x = A\sin\omega t$ (starting from mean):

$$t = \frac{1}{\omega}\sin^{-1}\left(\frac{x}{A}\right)$$

Using $x = A\cos\omega t$ (starting from extreme):

$$t = \frac{1}{\omega}\cos^{-1}\left(\frac{x}{A}\right)$$

Key time landmarks (starting from mean, $x = A\sin\omega t$):

Position	Time
Mean $\to A/2$	$T/12$
Mean $\to A/\sqrt{2}$	$T/8$
Mean $\to A$	$T/4$
$A \to -A$ (full swing)	$T/2$

8 Damped & Forced Oscillations

Damped amplitude: $A(t) = A_0\,e^{-bt/2m}$

Damped frequency:

$$\omega_d = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

- **Resonance:** Amplitude is maximum when driving frequency = natural frequency ($\omega = \omega_0$)
- Energy of resonance is maximum at $\omega = \omega_0$
- For damped oscillator with resonance: amplitude max at $\omega_1 \neq \omega_0$; energy max at $\omega_2 = \omega_0$

AIPMT 1998 PYQ:
Amplitude is max at $\omega_1 \neq \omega_0$
Energy is max at $\omega_2 = \omega_0$

9 Two Pendulums In Phase

If two pendulums of lengths L_1 and L_2 ($L_1 < L_2$) start in phase, they are next in phase at mean position when the shorter one has completed n oscillations:

$$n_1 T_1 = n_2 T_2 \quad \Rightarrow \quad \frac{n_1}{n_2} = \frac{T_2}{T_1} = \sqrt{\frac{L_2}{L_1}}$$

Minimum n_1 = LCM approach: $n_1 - n_2 = 1$

NEET 2022 PYQ: $L_1 = 100$ cm, $L_2 = 121$ cm
$\frac{n_1}{n_2} = \sqrt{\frac{121}{100}} = \frac{11}{10}$
$\Rightarrow n_1 = 11,\ n_2 = 10$ ✓

10 Quick Reference — Key PYQ Concepts

- Average velocity in one full oscillation = **0**
- Average speed in one full oscillation $= \frac{4A}{T} = \frac{2A\omega}{\pi}$
- Displacement of SHM is **zero** at mean; **max** at extreme
- Restoring force: $F = -kx$ (always towards mean)
- For $a = -\omega^2 x$: the coefficient of x gives ω^2
- Total distance in one time period $= 4A$; displacement $= 0$

Identifying SHM (PYQ 2007, 2009):
$y = \sin\omega t - \cos\omega t = \sqrt{2}\sin(\omega t - \pi/4)$ **IS SHM**
$y = \sin^3\omega t$ is **NOT** SHM (not of form $-\omega^2 x$)
$y = 1 + \omega t + \omega^2 t^2$ is **NOT SHM** (not periodic)

Angular SHM — Physical Pendulum:

$$T = 2\pi\sqrt{\frac{I}{mg\,d}}$$

where I = moment of inertia, d = distance of CM from pivot.

ω vs f vs T summary:
$\omega = 2\pi f = \frac{2\pi}{T}$

$T[\text{spring}] = 2\pi\sqrt{\frac{m}{k}}$

$T[\text{pendulum}] = 2\pi\sqrt{\frac{L}{g}}$

Energy summary:
$E = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2 A^2$
$v = \omega\sqrt{A^2 - x^2}$
$v_{\max} = \omega A, \quad a_{\max} = \omega^2 A$

Phase differences:
$x \to v$: leads by $\pi/2$
$x \to a$: phase diff $= \pi$ (opposite)
Freq of PE/KE $= 2n$ if SHM freq $= n$
Amplitude of superposition: $\sqrt{A^2 + B^2}$

All formulae verified against NEET/AIPMT PYQs (1998–2025). Focus heavily on sections 2, 3, 4, 5, 6.